Solving My Favourite Math Problem

Kai Hamann[1]

Writing across the University of Alberta, 2025[2]
Volume 6, pp. 45-52
Published December 2025
DOI: 10.29173/writingacrossuofa93

Introduction

"Solving My Favourite Math Problem" is a delightful narrative about how math can be taught in the form of a story. This piece was written for a WRS 104 course, in which students were able to choose any topic in order to create an argumentative research paper. Kai Hamann shows the readers how learning math can be exciting, creative and fun!

Keywords: math, Pascal's Triangle, puzzle, story

[1] Kai Hamann is a 2nd-year, Mathematics-major student at the University of Alberta.
[2] *Writing across the University of Alberta* (WAUA) publishes undergraduate student writing from writing studies courses and courses focused on writing studies practices and scholarship at the University of Alberta. You can find WAUA online at https://writingacrossuofa.ca/.

My favourite math problem is what happens when you try to take the infinite sum of whole numbers. Whole numbers refer to numbers like 1, 2, 3, 100, etc… The infinite sum looks something like 1+2+3+4+… The natural conclusion would be that since you are adding increasing numbers each time, the sum should be infinity or at least a positive number. But interestingly, the correct answer to this infinitely long sum is -112. Not only is the answer somehow a fraction, but it is a negative number. How can this possibly be true? Solving this mystery requires substantial background knowledge in math that may seem unrelated to the problem.

As a future math teacher, I'm wondering how I can create this sense of mystery for my future students. In this essay, I want to explore ideas to better engage people in math. And I want to walk you, the reader, through the solution to my favourite math problem.

Math as a Puzzle

Although I'm a math major now, I wasn't always interested in math. When I was in grade seven, I went to a school for kids with learning disabilities because of my dyslexia and ADHD. In math class, I had to do 30 minutes of work on a website called MathFactsPro. The program asked simple, repetitive math facts; it was designed to help students get the basics down. While it probably helped the people who needed extra practice in math, I hated it. It turned math into a dull, uninteresting, and repetitive subject. I even wrote an essay on why doing MathFactsPro was a waste of time when I was supposed to be focusing on my math facts. It should say something that the dyslexic kid would rather write an essay than do math problems.

The following year, I was homeschooled during the pandemic. My dad and I started watching math videos online, specifically Khan Academy and 3Blue1Brown. This is where my love of math began. My dad and I would look through all of the Khan Academy lessons in physics or math, and my dad would ask me which lesson I wanted to watch. I would pick the lesson that interested me the most, even if it was far above my grade level. We would watch the lesson, and it would upset me that I didn't understand anything. However, my dad would determine what I needed to learn to understand the lesson, and we would work through the concepts one by one. Throughout the process, I knew why I was learning what I was learning. It was like solving a puzzle. We slowly saw the bigger picture as we added each piece, even when there were still missing pieces. I loved this style of learning, and I was motivated to learn.

Pascal's Triangle

As I introduced at the beginning of this essay, the infinite sum of whole numbers is almost impossible to prove by attempting to solve the problem directly. Instead, you need to look at patterns that are seemingly unrelated to the puzzle at hand, patterns like Pascal's Triangle. Named after the 17th-century French mathematician Blaise Pascal, the triangle is a series of brick-like rows where each number is the sum of the two numbers above it (see Figure 1). For example, if you look at the digit 3, the two numbers above are 1 and 2, and 1 + 2 = 3. This pattern applies to every square. On either side of the triangle, we add zeros for consistency.

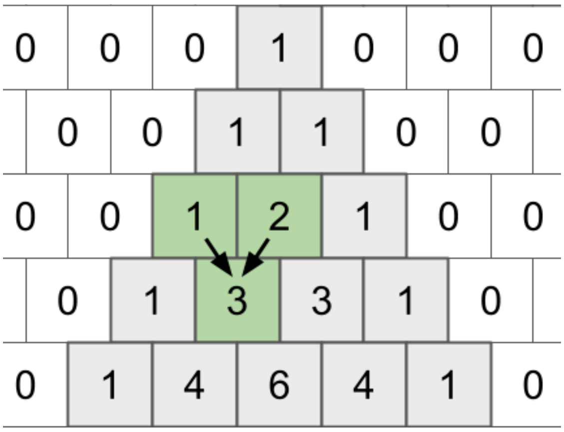

Figure 1. Intro to Pascal's Triangle

Another thing you might notice about Pascal's Triangle is that the sum of each row seems to double every time you go down a row. As you can see in Figure 2, rows are labelled starting from 0. The first row is Row 0, the second is Row 1, etc. We can use the expression 2n, where n is the row number, to show the pattern of how the sum of the numbers in each row doubles. For example, in the third row, n = 2, the sum is 22 = 2×2 = 4.

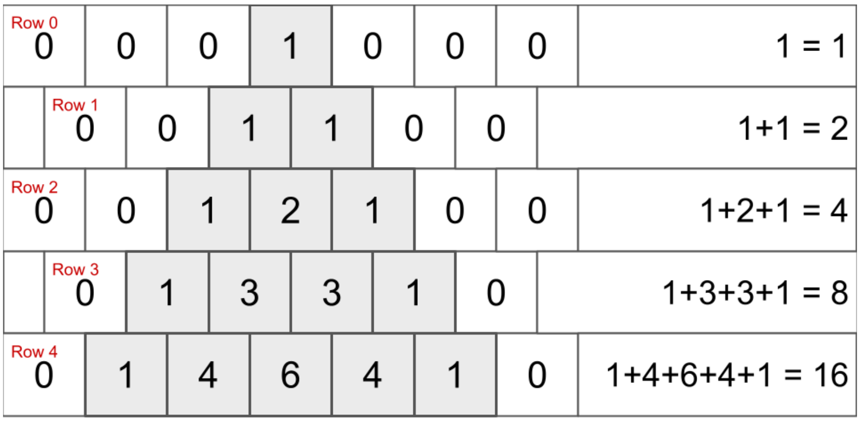

Figure 2. Sums of Pascal's Triangle

The Problem with High School Math

As someone interested in getting people engaged with math, I have to consider one of the most common questions in math: "When will I ever use this?" In elementary school, basic math skills like addition, multiplication, and reading graphs help us communicate in a modern society, making it easy to justify the need to learn these concepts. When it comes to high school and more advanced math, teachers may struggle to find examples of applications that work for everyone. For example, a student might not believe that learning Pascal's Triangle will ever be useful. (You probably don't, either!)

Part of the problem is the structure of the curriculum. Instead of creating a curriculum designed to foster intrinsic motivation in students, units are placed to fit neatly with one another to optimize space and efficiency, like a game of Tetris; for example, the math curriculum of Alberta (Government of Alberta 2025, 13). Dividing the curriculum into these neat units may make it easier to teach according to a set plan. This approach encourages teachers and students to view math as a series of small, digestible pieces. For example, in a study about math teachers in Alberta, one teacher said, "I thought that's what my job was–to make math simple into little bits so that students could consume it and regurgitate it" (Chapman, 450). When ideas are simple, students may be less engaged because the ideas don't have nuances to be explored. Simple questions are like low-quality digital images—they take up less space, but they communicate less about their subject. Complex questions are like high-quality images that convey the subject more completely, even if you don't recognize all the nuances.

To engage students better, I believe we should restructure the high school math curriculum around a few questions that can be explored in detail. Applied math and physics problems are a natural place to center these questions, as most of these problems require a deep understanding. To solve the puzzle, students need to understand these concepts, which attaches a justification for learning them.

By having the units focused on complex questions, students would have a clear goal in the present and become curious to solve the problem. Like a maze, there might only be one narrow correct path to the answer, but if we explore all the dead ends, where the correct answer is the climax of the unit, we can cover broader curricular content. However, this all hinges on the question being deep enough and interesting enough to engage students. What kinds of questions will engage students?

Above the Triangle

Let's ponder Pascal's Triangle some more. Instead of going from top to bottom, what happens if you go from bottom to top? You might find that for Row 2, which has 1, 2, and 1, you can figure out what goes above it. Starting at the left edge of the triangle on Row 2, we know the left 1 is the sum of 0 + 1 (See Figure 3). Can you do this process for Row 0, though? What goes above 1?

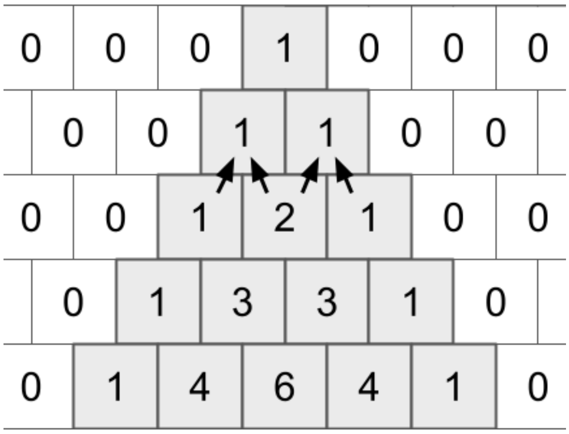

Figure 3. Reverse of Pascal's Triangle

If you use the same process, you will find that the numbers above 1 in Row 0 must be 0 and 1. However, as we move to the right on Row 0, we have to add negative numbers to make the pattern continue. The square next to the 1 in Row 0 is zero, so whatever is above it must add up to zero. Since we know that one of the numbers must be 1 for the 1 square to be true, we deduce the next number must be negative 1 because this is the only way for the number next to 1 to be a zero. In math language, we would write this as 1 + x = 0. We then solve for x and get x = -1. This pattern of alternative positive and negative numbers must repeat in order to cancel each other, leaving zero in Row 0. If we continue to add negative rows, we get the pattern you see in Figure 4. Does Row -2 (1-2+3-4+...) in Figure 4 seem familiar? It looks close to the original problem that we want to solve: 1+2+3+4...

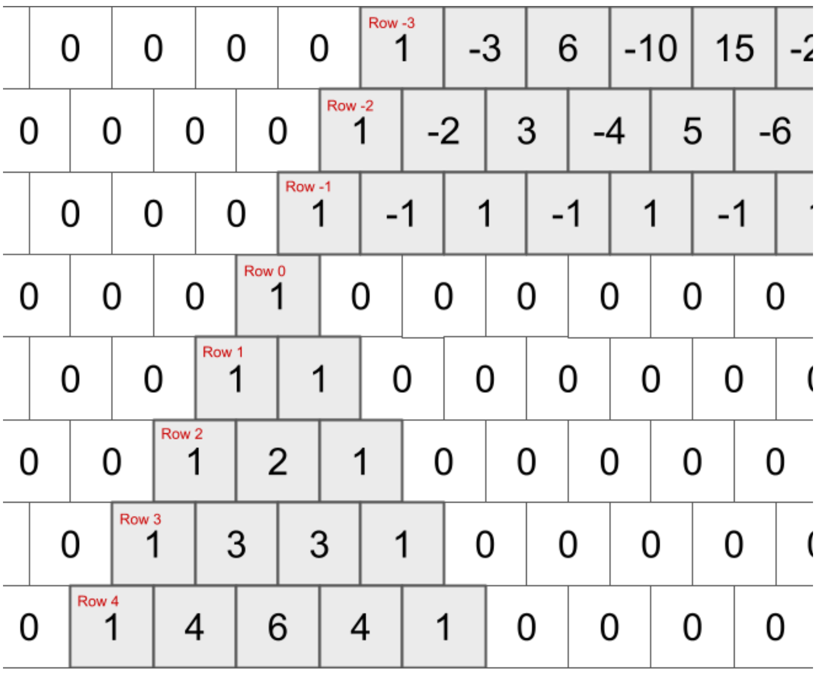

Figure 4. Negative Rows of Pascal's Triangle

Let's Tell Math Stories

Focusing on a complex problem in applied math or physics helps answer the question, "When will I use this?" It establishes why a concept is useful from the start. It attaches clear motivation and application to what students are learning. This approach may elevate math out of being an exclusively nerdy subject; it just feels good to do something productive.

However, this is not the only way to get someone engaged with math. Another way is through story, which is a way to organize a complex problem. Professional mathematicians often describe math as science fiction, meaning we play with ideas that can't be experienced in the real world. But bizarre ideas alone are not engaging; you need a narrative, and that is the role of a central question. It adds a story where students are possibly drawn into the bizarreness of the puzzle. For instance, the infinite sum of whole numbers resulting in a negative fraction is weird. But by taking it step by step and introducing the key insights of the puzzle, solving the problem becomes approachable. I want students to believe that they could be the first to solve such a puzzle.

A well-known math YouTuber, Grant Sanderson, discussed this in his TED Talk, "What makes people engaged with math?" He proposes that the key to getting people engaged with mathematics is not only application but also storytelling. To interest people in math, he argues we need topics "that pull you [into] the math for what it is now, not what it promises to give you later" (Sanderson, 11:21). Applied problems in math and physics often fail to do this; we need problems that make us ask questions as a snap reflex. This is what puzzles like our infinite sum do; if you have a soul, you have to know why the sum of infinite whole numbers is a negative fraction. It challenges our understanding and develops us as problem-solvers and critical thinkers. In addition, it uses known pedagogical principles like the bizarreness effect, "which refers to the ability to better recall and remember events or items that are unusual, uncommon, and distinct" (Basile et al. 129). Curiosity will drive students to gain a deeper understanding of math.

My experience learning math with my dad introduced me to the thrill of curious properties and connections in math. However, it also revealed some of the issues with the way we currently teach math. I want to share the beauty and marvel that math can evoke, and hopefully, you can believe that you may have been the one to show 1+2+3+4+... = –112.

Solving the Puzzle

The last thing we need to show why 1+2+3+4+... = –1/12 is to find the sums of the rows above Pascal's Triangle. For the row at n = –1, the process of finding the sum of the terms is the same as in the triangle, just a little more mathematical. If we add S_1 to itself, each term except for the first cancels itself out, leaving us with $2S_1 = 1$. Hence $S_1 = 1/2$.

$$S_1 = 1 - 1 + 1 - 1 + \ldots$$

$$S_1 + S_1 = \begin{array}{r} 1 - 1 + 1 - 1 + \ldots \\ \underline{+ 1 - 1 + 1 - \ldots} \\ 1 + 0 + 0 + 0 + \ldots \end{array}$$

$$2S_1 = 1$$

$$S_1 = 1/2$$

The same process for the row n = -2 as adding it to itself leaves us with the sequence 1-1+1-1+... = S_1 = 1/2, so dividing both sides by 2 leaves us with S_2 = 1/4. Another fun pattern is that the sum is still equal to 2^n since $2^{-1} = ½$ and $2^{-2} = ¼$.

$$S_2 = 1 - 2 + 3 - 4 + \ldots$$

$$S_2 + S_2 = \begin{array}{r} 1 - 2 + 3 - 4 + \ldots \\ \underline{+ 1 - 2 + 3 - \ldots} \\ 1 - 1 + 1 - 1 + \ldots \end{array}$$

$$2S_2 = S_1 = 1/2$$

$$S_2 = S_1/2 = 1/4$$

Finally, relating these to our original puzzle. This one doesn't follow the same rules as the other ones, so it takes a bit more tinkering, but eventually, you might find that taking 1+2+3+4+... and subtracting 1-2+3-4+... cancels out every other term, leaving us with a sequence 4+8+12+16+... which is precisely four times 1+2+3+4+... Hence, solving for S, we find 1+2+3+4+... = -1/12.

$$S = 1 + 2 + 3 + 4 + \ldots$$

$$S - S_2 = \begin{array}{r} 1 + 2 + 3 + 4 + \ldots \\ \underline{- \; 1 + 2 - 3 + 4 - \ldots} \\ 0 + 4 + 0 - 8 + \ldots \end{array}$$

$$S - S_2 = 4 + 8 + 12 + 16 + \ldots = 4S$$

$$-3S = S_2$$

$$S = -S_2/3 = -1/12$$

Works Cited

Alberta Education. "Mathematics Grades 10 to 12 Program of Studies." *Education, Government of Alberta*, 23 Nov. 2015, https://education.alberta.ca/media/564028/math10to12.pdf. Accessed 11 April 2025.

Basile, Elizabeth, et al. "Connecting weirdness and wonder to mathematics." *South Carolina Association for Middle Level Education Journal*, vol. 1, 2022, pp. 128–131, https://scholarcommons.sc.edu/scamle/vol1/iss1/15/. Accessed 4 April 2025.

Chapman, Olive. "Understanding change through a high school mathematics teacher's journey to inquiry-based teaching." *Journal of Mathematics Teacher Education*, vol. 13, 2010, pp. 445—458. Springer Nature Link, https://link.springer.com/article/10.1007/s10857-010-9164-6. Accessed 10 April 2025.

Sanderson, Grant. "What Makes People Engage With Math - TEDxBerkeley." YouTube, 13 March 2020, https://www.youtube.com/watch?v=s_L-fp8gDzY. Accessed 11 April 2025.